# Chapter 2.2 Equations of a Line

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• **Slope**: Slope can be expressed in a variety of ways including:

$$m = \frac{change \quad in \quad y}{change \quad in \quad x} = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

**Example 1.** Find the slope of line going through (-6,8) and (5,4)

Make one of the order pairs say (-6,8) your $x_1$ and $y_1$ and make your second ordered pair (5,4) your $x_2$ and $y_1$ and plug into the slope equation above.

$$m = \frac{4-8}{5-(-6)} = \frac{-4}{11}$$

• Slope of a horizontal line is 0

• Slope of a vertical line is undefined

• **Slope-intercept form**: Given a slope, m, and a y-intercept, b, the slope-intercept formula is

$$y = mx + b$$

**Example 2.** Find the equation of the line with y-intercept $\frac{7}{2}$ and slope $\frac{-5}{2}$

All you need to do is plug in the y-intercept value for b in the equation above and the slope for m in the above equation you get

$$y = \frac{-5}{2}x + \frac{7}{2}$$

**Example 3.** Find the equation of a horizontal line with y-intercept 3

In this case a slope was not given but since the line is a horizontal line we know the slope is zero so we get that the equation is:

$$y = 0x + 3 \rightarrow y = 3$$

**Example 4.** Find the slope and y-intercept of the line $5x - 3y = 1$

Easiest way to find the slope and y-intercept is to rewrite the equation in slope-intercept form.

$$5x - 3y = 1$$
$$-3y = -5x + 1$$
$$y = \frac{5}{3}x - \frac{1}{3}$$

We then get that the slope is $\frac{5}{3}$ and that the y-intercept is $-\frac{1}{3}$

• **Parallel and perpendicular lines**: Parallel lines are lines that have the same slope. Perpendicular lines are ones whose slopes are opposite reciprocals of each other. Another way to tell if lines are perpendicular is if when you multiply the two slopes together you get -1.

**Example 5.** Determine whether the pair of lines is perpendicular, parallel, or neither.

$2x + 3y = 5$ and $4x + 5 = -6y$

Easiest way to tell is by putting them in slope-intercept form and comparing slopes.

$$2x + 3y = 5 \to y = -\frac{2}{3}x + \frac{5}{2}$$
$$4x + 5 = -6y \to y = -\frac{2}{3}x - \frac{5}{6}$$

Looking at the slopes we see that both of the lines have a slope of $-\frac{2}{3}$ and therefore the lines are parallel.

**Example 5.** Determine whether the pair of lines is perpendicular, parallel, or neither.

$3x = y + 7$ and $x + 3y = 4$

$$3x = y + 7 \to y = 3x - 7$$
$$x + 3y = 4 \to y = -\frac{1}{3}x + \frac{4}{3}$$

Looking at the slopes we see that both of the lines don't have the same slope so they aren't parallel. Multiplying 3 and $-\frac{1}{3}$ we get -1 so the lines are perpendicular.

• **Point-Slope Form**: Given the slope of a line and a point on that line we can write an equation and get point-slope form defined as:

$$y - y_1 = m(x - x_1)$$

**Example 6.** Find the equation of the line that goes through point (1,3) and has a slope of 2

Plugging in 1 for $x_1$, 3 for $y_1$, and 2 for m we get that the equation is

$$y - 3 = 2(x - 1)$$

While this is technically correct most homework responses some answers on the test will start like this and ask you to write it in slope-intercept form. To do this just solve for y, doing so gives us

$$y - 3 = 2(x - 1)$$
$$y - 3 = 2x - 2$$
$$y = 2x - 5$$

**Example 7.** Find the equation of the line that goes through points (5,4) and (-10,-2)

In this case we need to find the slope first and then use either of the points to write the equation of the line in point-slope form. Let either ordered pairs be $(x_1, y_1)$ and the second order pair be $(x_2, y_2)$ In this case I will let (5,4) be $(x_1, y_1)$ and (-10,-2) be $(x_2, y_2)$ we get then that the slope is

$$\frac{-2-4}{-10-5} = \frac{6}{15} = \frac{2}{5}$$

We can then pick either order pair to write the equation they will look differently but will both be correct. If you would write each in points slope you would get the same equation

$$y - 4 = \frac{2}{5}(x - 5)$$

$$y + 2 = \frac{2}{5}(x + 10)$$

• We saw earlier that the equation of a horizontal line looks like y = b where b is the y-intercept. Similarly, the equation of the vertical like takes the form x = k where k is the x-intercept.

**Example 8.** The world-wide sales (in billions) of men's razor blades can be approximated by the equation $y = .76x + 4.44$ where $x = 6$ corresponds to the year 2006

a) Where were the approximate razor sales in 2011?

For this we need to find the number of razor sells which is y in our equation. To find this use 11 for x and we get

$$y = .76(11) + 4.44 = 12.8$$

so the number of razor blades sold is 12.8 million.

b) In what year did sales reach $10.5 billion?

Here we are looking for the year so we are looking for x, plugging in 10.5 for y and soling we get

$$10.5 = .76x - 4.44$$

$$6.06 = .76x$$

$$x = 8$$

This means that the year will be 2008.